Procedural Techniques for Simulating the Growth of Plant Leaves and Adapting Venation Patterns

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Figure 1: *Our system allows for the simulation of various types of leaves including different venation patterns and complex leaf shapes*

Abstract

This paper presents biologically-motivated a procedural method for the simulation of leaf contour growth and venation development. We use a mathematical model for simulating the growth of a plant leaf. Leaf tissue is regarded as a viscous, incompressible fluid whose 2D expansion is determined by a spatially varying growth rate. Visually realistic development is described by a growth function $\mathbb{RERG}$ that reacts to hormone (auxin) sources embedded in the leaf blade. The shape of the leaf is determined by a set of feature points within the leaf contour. The contour is extracted from photos by utilizing a Curvature Scale Space (CSS) Corner Detection Algorithm. Auxin transport is described by an initial auxin flux from an auxin source to an auxin sink that is gradually channelized into cells with high levels of highly polarized transporters. The leaf is presented as a triangulated double layer structure that consists of a Voronoi-Diagram that is discretised along the vein structures.

CR Categories: I.3.3 [Computer Graphics]: Three-Dimensional Graphics and Realism—Display Algorithms I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Radiosity;

Keywords: Leaf development, auxin rate, curvature scale space(CSS), animation, botanical simulation

1 Introduction

Realistic modeling of growing plant leaves has received only a limited attention in computer graphics. This is interesting since it also

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has a long history in bridging biology, theoretical studies of morphogenesis, and visualization[Prusinkiewicz 1994]. In this paper a mathematical model is presented to simulate the growth of leaves. The tissue of the leaf is regarded as a viscous, incompressible fluid whose 2D expansion is caused by a non-zero growth rate that differs locally [Wang et al. 2004]. We propose a physically-based approach to model growth based on the corresponding expansion rate [Coley et al. 2006]. We ignore body forces such as gravity and only assume surface forces.

Leaf form and vascular patterns provide some of the most impressive examples of the complexity of biological shapes generated in nature. In multicellular organisms, boundaries have the role of preventing the intermingling of two different cell populations and in organizing the morphogenesis of organs and the entire organism. Plant leaves have two different cell populations, the adaxial (or upper) and abaxial (or lower) cell populations, and the boundary is considered to be important for lamina growth [Nakata and Okada 2013].

In this paper we will character the growth by growth tensor field at $\mathbb{X}$ and $\mathbb{Y}$ and $\mathbb{Z}$ coordinates [Hejnowicz and Romberger 1984; Nebelsick et al. 2001; E et al. 2004]. The growth tensor allows full characterization of the rate of growth in length, area, and volume, as well as rates of angular change between elements, and of vorticity in the growing organ. The growth tensor is a generalization of the relative elementary rate of $\mathbb{X}$ Coordinate growth $\mathbb{RERG}_x$, $\mathbb{Y}$ Coordinate growth $\mathbb{RERG}_y$, and $\mathbb{Z}$ Coordinate growth $\mathbb{RERG}_z$.

Furthermore, the morphology of leaves is highly determined by wide variety in leaf venation patterns that has been classified for example in [Nebelsick et al. 2001]. We use this system and adopt its terminology. This classification system does not only consider the geometric arrangement of different vein classes but also their relation to other architectural features of leaves.

Figure 2 gives an overview of our proposed method. We simulate the interplay between different processes and use a mathematical model to simulate the growth of a plant leaf. *(i)* : we developed a method for classifying a plant leaf by utilizing features of the leaf contour. A simple image of the contour can be extracted by utilizing a Curvature Scale Space (CSS) Corner Detection Algorithm. *(ii)* :

A model generates visually realistic development using the growth function RERG that lets the leaf grow towards sources of the hormone Auxin which are embedded in leaf blade. The growth rate of plant organ a scalar of a quantity vector, that depends on the level of rigor with which we scrutinize our perception of growth. *(iii)* : a auxin transport canalization-based model describes a process in which an initial auxin flows from a source to a sink while it is gradually canalized into files of cells with high levels of highly polarized transporters. Our model is expressed in geometric terms and uses proximity criteria to determine new vein locations. *(iv)* : The leaf is presented as a triangulated double layer structure that consists of a Voronoi-Diagram discretized along the vein structure and its corresponding diagram. The leaf mesh is mapped to deformed saddle-like mid-surface and rippled contour.

2   Related Work

Related work for our approach encompasses biology as well as modeling methods in computer graphics.

Biological Modeling: In literature, repeatedly a basic module was used to simulate the development of common feature of the leaf lamina and vein networks. The vascular system of plants consists of a network of cell files (vascular strands) that extends through all organs [Scarpella et al. 2010; AG and P 2005; Bilsborough et al. 2011]. Over the past 20 years, genetic approaches have led to substantial increase in our understanding of leaf and vascular development, and have provided good evidence that the growth regulator auxin provides important spatial cues for this. Since inhibition of auxin transport affects the formation venation patterns, auxin is likely to be part of the involved signal [Scarpella et al. 2010]. In this paper we want to utilize auxin for the visual formation of plant leaves.

Haiyi et al. [Liang and L 2009] produce a model that uses geometric and growth control parameters to determine the shape of finite laminae. This allows for a comparative study of elongated leaf morphology. In [Liang and L 2009] a shape space for a growing elastic leaf is designed by using a combination of scaling concepts, stability analysis, and numerical simulations. This combination is considered as increased relative growth strain. A long flat lamina deforms to a saddle shape and/or develops undulations that may lead to strongly localized ripples as the growth strain is localized to the contour of the leaf. Haiyi et al. [Liang and Mahadevan 2011] use a combination of surgical manipulations and quantitative measurements to confirm this hypothesis and provide a simple theory for changes in the shape of a doubly curved thin elastic shell subject to differential growth across its plan-form. This functional morphology suggests new bio-mimetic designs for deployable structures using boundary or edge actuation rather than the usual bulk or surface actuation.

Hang et al. [Xiao and Chen 2011] establish phenomenological buckling models to explain the curled configuration of dried leaves, where the driving force is the differential contraction strain field. In the minimalist model, through a systematic study, the averaged buckling curvature is correlated with the aspect ratio and normalized size of the leaf, as well as the magnitude of the differential strain. Hong et al. [ming Xu and jian He 2012] propose a method for modeling curly plant leaves that is based on venation skeletons driving leaf surface deformation. A 2-D leaf silhouette was extracted from a scanned leaf image. The algorithm computed the primary veins (medial axes) of a leaf, along which secondary veins were branched out automatically. SoHyeon et al. [Jeong et al. 2013] simulates the whole leaf surface to capture the fine details of desiccated leaves. In this paper we will present the whole leaf surface as a triangulated double layer structure that consists of a Voronoi-Diagram discredited along the vein structure corresponding diagram. The leaf mesh is mapped to deformed saddle-like mid-surface and rippled contour.

In Computer Graphics, different approaches have been proposed for modeling plants leaves: image-based modeling, particle systems, implicit contours, and L-systems [Peyrat et al. 2008].

Image-based modeling: a first paper for modeling leaves uses image-based modeling. Such methods reproduce a real shape of a leaf provided by the user. Quan et al. [Quan et al. 2006] use a semi-automatic technique for modeling plants leaves directly from images. The approach has the advantage that the resulting model inherits the realistic shape and complexity of a real plant. The user provides several pictures taken from different angles, then a point cloud is built and the segmentation of individual leaves (via a graph) is done with the image information. After this the user can manually refine the segmentation in order to bypass errors due to overlapping leaves. A generic and deformable leaf model is then applied based on this information.

Particle systems: Rodkaew et al. [Rodkaew et al. 2004] propose a particle transportation algorithm for modeling plants in different colors and with complex venation structures. The algorithm is initiated by randomly scattered particles inside the blade of a leaf. Each particle contains energy. A transportation rule directs each particle toward a target. When particles are in close proximity, they are combined. The trails of moving particles are used to generate the venation patterns. Runions et al. introduced a biological algorithm in order to build leaves [Runions et al. 2005]; they use the shape of the leaf, then build veins via simulation of hormone distribution inside the leaf.

Implicit contours: a method designed by Hammel et al. in [Hammel et al. 1992] models compound leaves using implicit contours. This model creates a planar scalar field in the proximity of the skeleton. The margin is represented by a contour, defined as the locus of points with a given field value. The area bounded by this contour forms the surface of the leaf. Mundermann et al. in [Mundermann et al. 2003] design a method for modeling lobed leaves. This method extends the concept of sweeps to branched skeletons.

L-systems: The first methods to simulate certain patterns in nature found in plant development are L-systems [Prusinkiewicz and Lindenmayer 1996; Rodkaew et al. 2002]. These methods are very efficient to simulate plants organs and branching structures. Prusinkiewicz and Lindenmayer in [Prusinkiewicz and Lindenmayer 1996] proposed many methods based on L-systems. In [Terraz et al. 2009] an extension of L-systems is proposed, based on three-dimensional (3D) generalized maps that allow an easier control of the internal structure of 3D objects.

3   Simulation of Leaf Growth

Photosynthesis is a process used by plants and other organisms to convert light energy coming from the sun into chemical energy that can be later released to fuel the organisms' activities. In order to obtain realistic leaf growth, we develop a system using five processes to simulate leaf and leaf growth. The system is summarized in Figure 2.
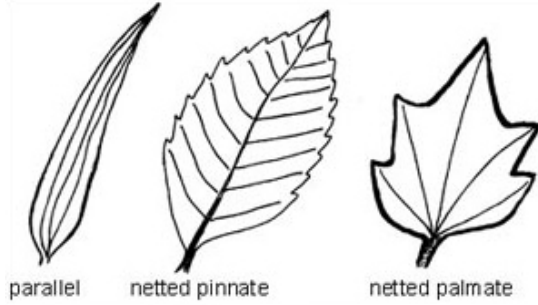
3.1   Leaf Type Modeling

The shape of the leaf blade and the type of leaf margin are important characteristics that help to identify plants (see Figure 3). Leaf blades vary to a large extent, they may be simple (apple, oak)
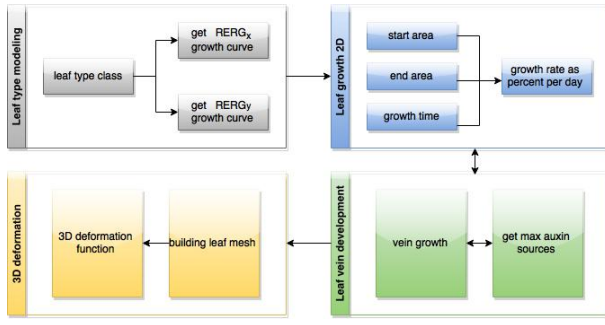
Figure 2: *Our system for leaf development.*

or compound (divided into several smaller leaf like segments, as in honeylocust). The smaller segments are called leaflets and are attached to a stalk (rachis) with a petiolule. Leaflets can also be arranged palmately (horse chestnut) or pinnately (ash). Pinnately compound leaves are said to be odd pinnate (ash) when ending in one leaflet and even pinnate when ending in two leaflets (locust). This terminology is important in identifying plants by their leaves.


Figure 3: *Different leaf shapes found in nature.*

Figure 2 shows the schematic modeling processes for our leaves. Our modeling system provide the user with the ability to select one leaf type from various given leaf classes. After that, the modeling processes start to get the 2D silhouette of a leaf from a given input photography which is derived from the selected leaf type. Then the relative elementary axis-aligned growth rates $\mathbb{RERG}_x$ and $\mathbb{RERG}_y$ are computed using the 2D silhouette. The relative elementary growth rate $\mathbb{RERG}_{l(\zeta)}$ is the rate at which an infinitesimal distance $\Delta\zeta$, measured in the direction of growth line at a point on growing silhouette, increases over time. $\mathbb{RERG}$ is mathematically defined as [Hejnowicz and Romberger 1984]:

$$RERG_{l(\zeta)} = (\frac{1}{\Delta\zeta})(\frac{d\Delta\zeta}{dt}) \qquad (1)$$

Using the relative elementary growth rate we create a growth curve (vector) for the $\mathbb{X}$ and $\mathbb{Y}$ coordinates of each leaf type (see Figure4). The growth rate of a leaf on $\mathbb{X}$ or $\mathbb{Y}$ is a scalar or a vector quantity. In order to produce different growth curves (vector) for same leaf type we used merge algorithm between two growth curve(vector) which belong to same leaf type. For example, when i have two growth rate vector $\mathbb{RERG}_{x1}$ and $\mathbb{RERG}_{x2}$ for one leaf type, can i get new growth rate vector $\mathbb{RERG}_{xn}$ by apply a merge (inheritance) algorithm on both vector $\mathbb{RERG}_{x1}$ and $\mathbb{RERG}_{x2}$.
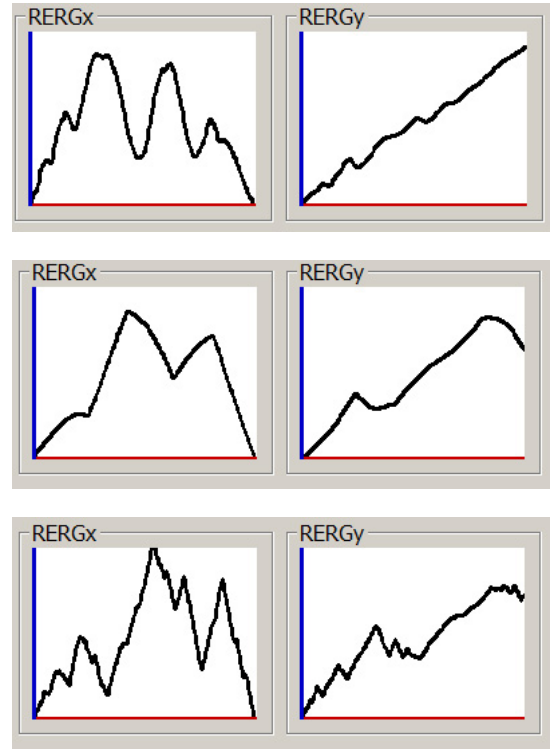
Figure 4: *Axis-aligned growth curves (vector) that define a leaf.*

3.2 Leaf Expansion

To simulate leaf growth we use a two-dimensional model since the thickness of a leaf can be considered negligible compared to the growth of its surface. Under these assumptions, the surface Expansion can be modeled by expansion rate as percent per day GR [Coley et al. 2006].

$$GR = 100 * [e^{(ln(Sarea/Earea)/t)} - 1] \qquad (2)$$

where $Sarea$ and $Earea$ are leaf area at two different measurements and t equals the number of days between measurements. The growth model used in this paper has been tested on the growth of several types of leaves. We observed that various parts of the leafs lamina expand at different rates, depending on their distance from the tip and the age of the leaf. Figure 5(a) illustrates the growth of a natural leaf while in 5(b)-(e) computer simulations are shown using the expansion rate GR. Our results faithfully resemble the natural growth.
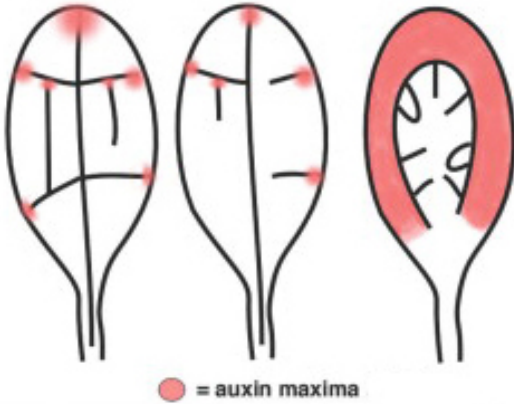
3.3 Determination of Auxin Sources and Maxima

Our algorithm distributes Auxin sources at leaf edges and then we simulate the venation process based on the Auxin level and consumption. Strong auxin sources are assumed to exist at edge locations that grow stronger than others [Stanko et al. 2014]. Figure 6 illustrates the relationship between the venation patterns and maximum Auxin sources.

In this paper, we used the Shi-Tomasi corner detector and good features [Shi and Tomasi 1994] to track the max auxin placement at the leaf contour which we get from the input photography. The growth in each leaf corner must be larger than for other leaf loca-
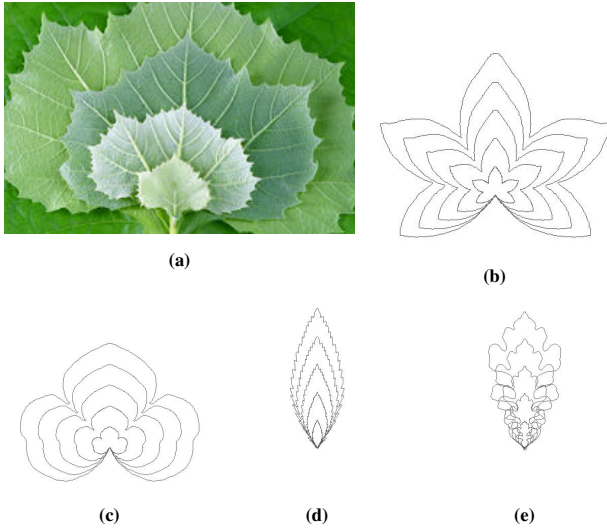
Figure 5: *Growth of a plant leaf in a time period. (a) Natural growth of a plant leaf. (b-e) Computer simulated growth of a plant leaf.*

Figure 7: *Maxima for the Auxin distribution in several leaf types.*

tions. Therefore we deduce Auxin maxima to be located at leaf corners. In Figure 7 the red and blue circle represent such sources.
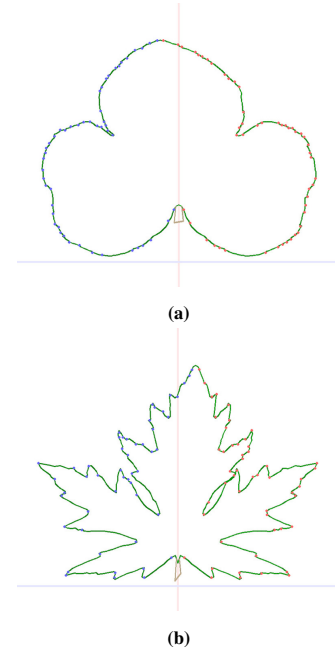

Figure 6: *Illustration of the relationship between auxin maxima and primary and secondary venation processes.*
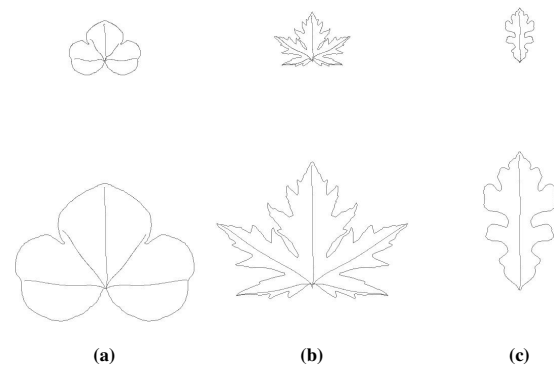
3.4 Generating and Growing the Venation System

The plant growth regulator Auxin seems also to be responsible for the development of the venation patterns [Scarpella et al. 2010]. For most leaves, the primary and secondary veins are not only the most obvious features but also play the most significant role in leaf deformation. Therefore in this paper we concentrate on simulating growth and development of primary and secondary veins. Our algorithm, however, can also simulate third and subsequent levels of veins.

During initial leaf growth, a small primordium becomes visible at the flanks of the SAM (the shoot apical meristem). Epidermal Auxin flow converges to form a maximum of Auxin activity at the tip of the primordium. It is drained through the center of the primordium, marking the position of mid vein of the new leaf. In
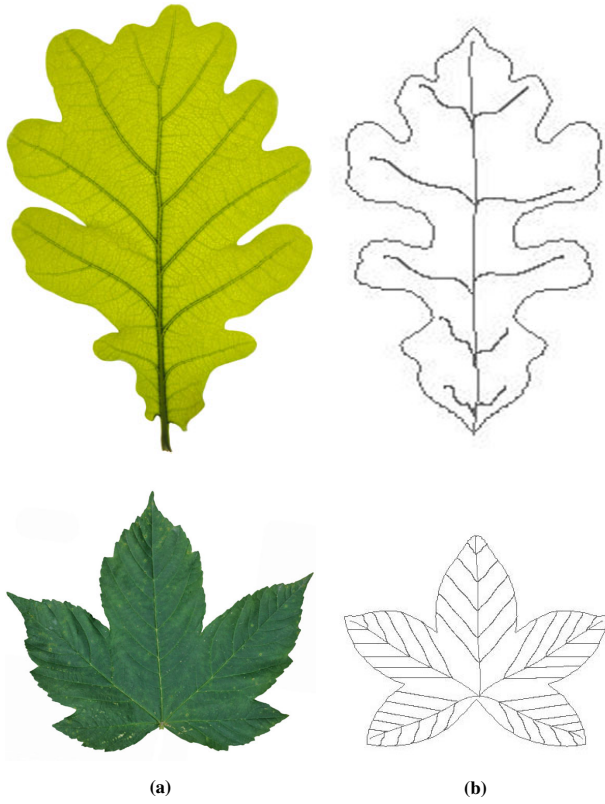

Figure 8: *Development of primary veins with respect to localization of max auxin. In first row illustrated canalization process for primary veins at first step. While the second row illustrated the development of primary vein after growth time.*
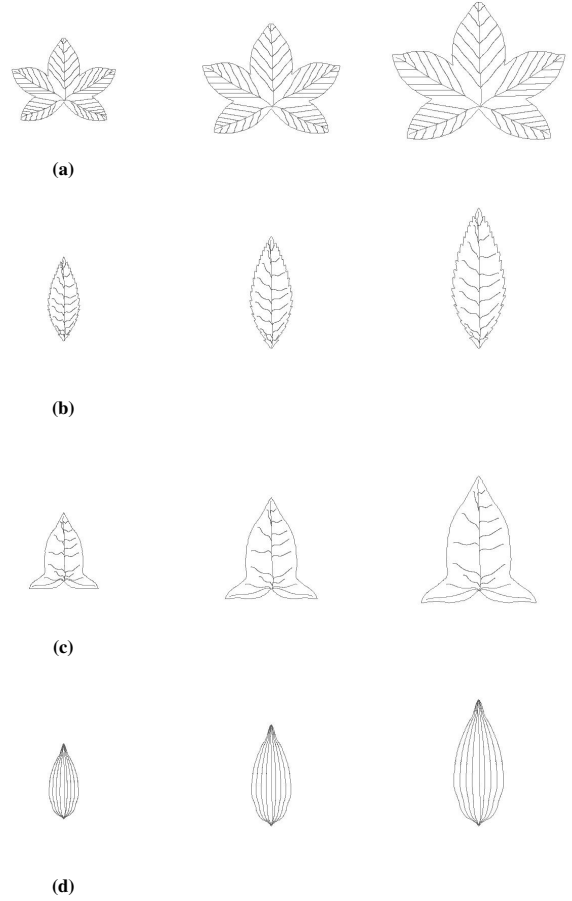
Figure 9: *Illustration of natural venation system and our venation system. (a) Natural leaf. (b) Computer simulated leaf venation.*

Figure 10: *Development of primary and secondary veins with respect to localization of max auxin for several leaf type.*

primary morphogenesis, leaves grow predominantly via cell division to acquire their shape and vascular pattern. Auxin maxima at the margins of the leaf correlate with sites of lateral vein formation and positions of serration development.
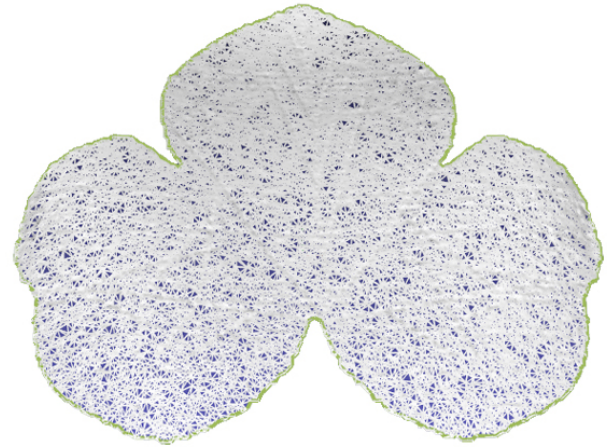
We generate the primary veins related to the given leaf type and the maximum of auxin sources placement. Maximum of auxin are assumed at locations that are growth rate farther than a threshold. Then canalized between maximum auxin and the nodes that are related on the leaf type. Figure 8 illustrates the development of primary veins with respect to the localization of maximum of Auxin at leaf margin.

In order to generate the secondary veins, we firstly develop a function to choose a parametric number n of nodes in each primary vein. Then we find n Auxin sources in the leaf margin. Each Auxin source is now assumed to join the vein node that is closest to it. In Figure 10 we simulate the development process of primary and secondary veins with respect to localization of Auxin maxima at leaf margin.

For our needs, we developed an interface for automatic generating venation systems from a leaf skeleton. The Venation skeleton is introduced for 2D shapes in order to provide a symmetry-based shape representation for perception and recognition, see Figure 10. As shown in Figure 9 the natural venation systems in column (a) and our computer simulated venation systems in column (b) are quite similar.

Figure 11: *Characterization of two layer leaf mesh.*

(a) **(b)**
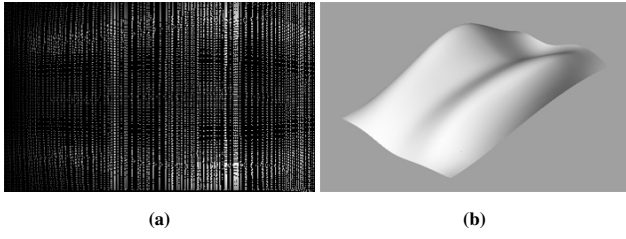
Figure 12: *Characterization of 2D and 3D deformation shape function.*

3.5 Building the Leaf Mesh

We have constructed a leaf skeleton with two boundary curves and midvein curve. To mesh the void area within these boundary curves, we employ Delaunay triangulation scheme, because it can deal with the problem of concave area in the leaf blade, see Figure 11. For example, lobed leaves often have irregular silhouette characterized by a number of concave outline.

In this paper each initial leaf is formed from two layers of meshes in order to regard the thickness of leaf. Generally, each vein-point is inserted as a new vertex in the leaf mesh in order to link mesh triangles with veins system. The leaf mesh is textured with different colors such that each color is related to the position and the type of the vertex. In addition to that, the leaf contour is prepared separately, and then it is linked to leaf mesh. The objective is to get more control on the deformation of the leaves contour and on their colors.

3.6 3D Deformation Function

Leaves and flowers usually have a typical, saddle-like mid-surface and a characteristic rippling pattern at their edges. To produce a perfect saddle shape we propose the function

$$\tau(x,y) = \lambda(x+y)^2 - \mu(x-y)^2,\qquad(3)$$

where $0 <= \lambda <= 1$ and $0 <= \mu <= 1$ are constants. It fulfills all the arithmetic conditions [Liang and L 2009], and its plot has the shape of a saddle, as in Figure 12. To investigate the mechanistic basis of rippling behavior and its physiological role during leaves growth, we use a function φ for periodic rippling [Liang and L 2009]:

$$\varphi(x,y) = \delta(y)sin(kx).\qquad(4)$$

Here k is the dimensionless wave number, and $\delta(y)$ is the cross-sectional profile of the surface.

In Figure 12 we show the resulting 2D and 3D deformation shapes from saddle and rippling functions. In order to get the 3D deformed leaf surface we utilize the deformation function illustrated in Figure 12. Our system can create several deformation shapes from different deformation function. In addition to that, each leaf type have its own deformation function.

4 Modeling Results

In order to implement our method we programmed an intuitive system using Microsoft Visual Studio 2013 c++, OpenCV, OpenGL, and QT library.

Firstly, we produced the growth curve for several leave types. Figure 4 shows different photos of growth curves used to simulate the growing of leaves. Eq. (1) represents a growth curve (DNA growth) on the x and y coordinates for each leaf type. the surface expansion

is modeled by expansion rate as percent per day GR, see Eq. (2). The growth of a plant leaf in a time period is simulated in Figure 5, which shows in (a) the natural growth of a plant leaf, and in (b-e) computer simulated growth of a plant leaf.

Our algorithm detects the placement of maximum auxin sources at leaf edges using Shi-Tomasi corner detector. Figure 7 illustrates the algorithm to determine the max auxin distribution for several types of leaves. After identifying the leaf type, max auxin placement, and number of primary veins we generate the primary venation development system for several leaf types. Figure 8 illustrates the development of primary veins with respect to localization of max auxin at leaf margin. Our system uses a sophisticated function in order to generate the secondary veins. The development of primary and secondary veins is simulated in Figures 9 and 10.

We mesh the void area within boundary curves using Delaunay triangulation scheme to generate 3D leaf form. In order to regard the thickness of leaf we simulate each leaf with two layers of mesh, see Figure 11. Our system creates several deformation shapes generated from many different deformation functions used to deform the leaf shapes, see Figure 12. Then we texture the leaf with different colors, see Figure 1.

Compared to previous related work on modeling and simulating growth of leaves, our method is motivated by all growth stage from biology and ecology science in order to get a natural modeling of several leaf development types.

5 Future Work

We presented a technique for constructing realistic leaf models for several leaf types; automatically, without any manual interactions. We showed how to model 2D natural growth for several types of leaves, and we illustrated 3D renderings of deformed results of each leaf Type. In the future we plan to develop our modeling system to include the simulation of different outer or inner effects on the leaves. Various effects, such as cracks, insect attacks, and growing spots, will be incorporated into the simulation to improve realism. Finally there are many natural phenomena that happen with leaves which we plan to simulate.

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