Tuti Weaving

Abdalla G. M. Ahmed* Oliver Deussen

Department of Computer and Information Science, University of Konstanz, Germany

Abstract

We present a new idea for the algorithmic design of weaving patterns that is based on AA Outlines. The method takes advantage of their inherent property of having only a few distinct rows and columns. In contrast to a prior Bridges paper, we describe a new approach for extracting weaving patterns from AA Outlines, and we add colors of yarns as a design parameter. The resulting patterns can be non-periodic, are free of long floats, and can be woven in looms with only two shafts.

Introduction

Weaving design is a good example of a topic that bridges math and art: the goal is to produce aesthetically pleasing woven patterns, and the way woven fabrics are structured and produced lends itself well to mathematical abstraction. Unsurprisingly, a large number of articles have been dedicated to the mathematics of weaving design. Ralph Griswold compiled a huge collection of literature of this kind [10], and contributed many articles by himself. Another prominent example is the website compiled by sarah-marie belcastro [5]. The Bridges conference also published some articles in this area, for example [2, 4, 8].

In this paper we contribute a new technique for algorithmic weaving design. We describe a new family of bicolor patterns that are readily weave-able in 2-shaft looms. This family is closely related to AA Bitmaps [4], but here we introduce a new design idea and furthermore we utilize yarn colors to reduce the number of shafts and to control floats. To motivate our proposed idea, we start with a brief discussion of weaving design and its challenges.

Weaving Design: Statement of the Problem

A simple woven fabric is a uniform mesh of yarns that run along the fabric, called the "warp," and yarns that run across, called the "weft." At each intersection, either the warp or the weft yarn runs above, and we see the color of that yarn, while the color of the yarn running beneath is seen on the other side. The whole piece of fabric can be abstracted as a two dimensional matrix with binary entries: each row corresponds to a weft yarn, each column corresponds to a warp yarn, and each entry indicates whether the warp yarn is (0) above or (1) beneath. For example, the weaving matrices for two common fabrics are:

$$\mathbf{S}_{\text{plain}} = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & \cdots \\ 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\ 0 & 1 & 0 & 1 & 0 & 1 & \cdots \\ 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\ 0 & 1 & 0 & 1 & 0 & 1 & \cdots \\ 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathbf{S}_{\text{twill}} = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & \cdots \\ 1 & 0 & 0 & 1 & 1 & 0 & \cdots \\ 0 & 0 & 1 & 1 & 0 & 0 & \cdots \\ 0 & 1 & 1 & 0 & 0 & 1 & \cdots \\ 1 & 1 & 0 & 0 & 1 & 1 & \cdots \\ 1 & 0 & 0 & 1 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

*Email: abdalla_gafar@hotmail.com

The weaving matrix '**S**' reflects the structure of the woven fabric, but not necessarily what is actually seen by the eye. Colors and other properties of individual yarns are superimposed on the weaving structure to determine the final look of the fabric, which could also be represented as a matrix '**F**'. Incorporating actual color information is a simple look-up process. A '0' in the weaving matrix is replaced by the color of the corresponding warp yarn, and a '1' is replaced by the color of the corresponding weft. Algebraically:

$$\mathbf{F}[i,j] = \mathbf{Clr_{weft}}[i].\mathbf{S}[i,j] + \mathbf{Clr_{warp}}[j].\bar{\mathbf{S}}[i,j]\,,$$

where $\bar{\mathbf{S}}$ is the complement of the weaving matrix; that is, 0's are replaced by 1's and 1's are replaced by 0's.

Choosing a weaving structure, along with properties of the yarns, is the subject of weaving design. It is a non-trivial task, since it is constrained by many factors. The weaving structure alone has two main constraints: one — the "floats" problem — relates to the fabric itself, and the other to the way it is produced. On the fabric side, if a warp or a weft yarn stays above or beneath for many steps without inter-weaving with the orthogonal yarns, it results in a weak fabric, and the yarn itself becomes prone to tearing and snagging. Floats are reflected as long runs of 0's or 1's in the weaving matrix; in rows (weft), columns (warp), or both. On the manufacturing side, woven fabrics are produced by a device called a loom; it weaves the fabric row by row, and for each row it raises the warp yarns designated to stay above (according to the weaving matrix) and passes the weft yarn beneath them. Weaving an arbitrary matrix requires full control over each individual warp yarn. This capability is provided by a so-called "Jacquard" loom, but such looms are very expensive. The alternative is to raise warp yarns in groups rather than individually. A so-called "dobby" loom uses so-called "shafts" to control groups of warp yarns. The more shafts a loom has, the more expensive it is. The smallest number of shafts to weave a given weaving matrix is equal to the number of distinct columns in the matrix.

On the other hand, choosing properties (color and texture) of individual yarns also has practical as well as operational constraints. Practically, the color of the yarn remains fixed along the whole row or column, which drastically limits the choices of weaving design in comparison to, say, tiling design: from a 2-dimensional matrix to a pair of 1-dimensional vectors. The operational complexities are related to the fitting of individual warp yarns to the designated shafts, which is known as "threading", and to the controlling of the raising sequence of the shafts, which is known as "treadling". A complex weaving matrix and/or a customized selection of properties of individual yarns raise the cost of these processes and make them more prone to mistakes.

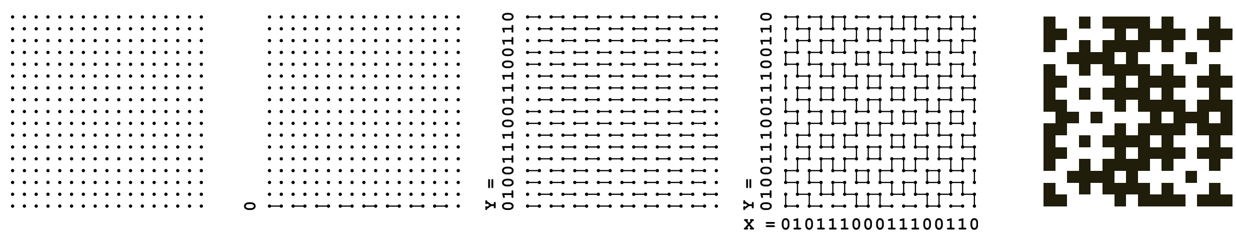
The problem of weaving design, then, is about producing visually pleasing woven patterns given all the aforementioned constraints. Specifically:

- The final pattern **F** should be visually rich.
- The weaving matrix **S** should not have long runs of 0's or 1's.
- **S** should have only a few distinct columns; 2 to 8, for example.
- It is preferable to have some pattern in **S** to simplify threading and treadling.

The problem is approached both intuitively, using the designer's sense and skills, and algorithmically, using mathematical formulation. Classical weaving design works by applying a pattern to threading and treadling sequences, and checking the appearance of the resulting weaving. Digital computers have made such a trial-and-error process much easier and quicker, as can be seen, for example, in Andrew Glassner's web-based interface [9]. Patterns can be designed by hand, as in the many cataloged patterns (see [6], for example), or derived from a mathematical formula, as in the works of Ada Dietz [7] and Ralph Griswold [10]. The whole approach, however, remains limited to small periodic patterns.

Algorithm 1 Drawing AA Outlines (reproduced from [4, Algorithm 1]).

1. Start with a uniform 2D grid of points. See Figure 1(a).
2. On the first row join every other pair of points, creating a dashed line. See Figure 1(b). We label this row 0 if the first and second points are joined and 1 if the second and third points are joined.
3. Use a binary sequence $\mathbf{Y}$ to label each row 0 or 1 and create the appropriate dashed line. See Figure 1(c).
4. Similarly use a binary sequence $\mathbf{X}$ to label each column 0 or 1 and create the appropriate dashed line. See Figure 1(d).



(a) (b) (c) (d) (e)

Figure 1 : *(a-d) Steps to build an AA Outline, and (e) the resulting AA Bitmap. Reproduced from [4, Figures 3 and 4]*

AA Bitmaps

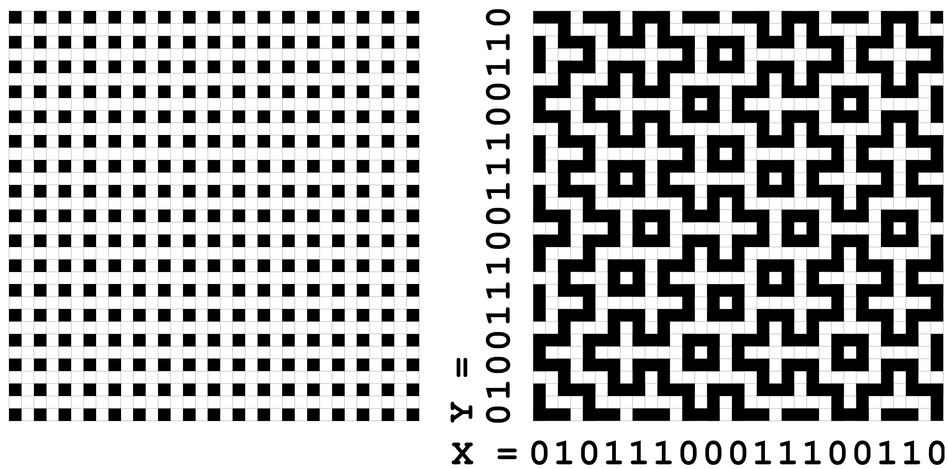
In 2013, Ahmed [4] presented a fundamentally different approach to the weaving design problem. Instead of tweaking a pattern to fit the loom, he used a family of algorithmic ornamental patterns — so-called AA Bitmaps — that inherently comprise only four distinct columns, and are therefore readily weave-able in a 4-shaft loom.. AA Bitmaps are obtained by first drawing so-called AA Outlines [1], as described in Algorithm 1, then painting these outlines in an even-odd fashion, as illustrated in Figure 1. The resulting pattern is made up of four distinct rows and four distinct columns, and an infinitude of periodic and aperiodic patterns can be obtained by changing the binary words $\mathbf{X}$ and $\mathbf{Y}$ that control the outlines.

Tuti Patterns

From the same AA Outlines we derive a new set of bitmaps that we call "Tuti Patterns," after a beautiful island at the junction of the two Niles (blue and white) in Khartoum, Sudan. Instead of painting between the outlines, we rasterize the outlines themselves. That is, we plot a grid in a bitmap, then connect the points as before. A bitmap is just a visual representation of a binary matrix. The minimal number of pixels to plot a grid is twice as much as the grid points[1], so the resulting bitmap is twice the size of the pattern; see Figure 2.

Let us inspect the structure of these patterns. Our convention is that 0 corresponds to black in the bitmap. Indexing the rows from 0, we can identify four distinct rows. The rows indexed $2i$ periodically read $[0y_i0\bar{y}_i]$, where y_i is the ith bit from the binary word $\mathbf{Y}$ that controls the rows. We use square brackets to indicate periodic repetition. The rows indexed $4i+1$ read $(x_01x_11x_21\cdots)$, and the rows indexed $4i+3$ read $(\bar{x}_01\bar{x}_11\bar{x}_21\cdots)$; that is, the word $\mathbf{X}$ and its complement $\bar{\mathbf{X}}$, respectively, padded with ones. Analogous

[1]This is known as the Nyquist rate, and it is a corner stone in Information Theory.



(a) (b)

Figure 2 : *A Tuti Pattern obtained by rasterizing (a) the grid in Figure 1(a), then (b) the outlines in Figure 1(d).*

arguments hold for columns, and as such these patterns, treated as weaving matrices, can be woven in a 4-shaft loom. As per the preceding description of rows and columns, the longest floats in even rows and columns are 3 steps. In contrast, depending on the run-lengths of 1's and 0's in $\mathbf{X}$ and $\mathbf{Y}$, arbitrarily long floats can be found in oddly indexed rows and columns. Thus, many Tuti Patterns might be unsuitable weaving matrices. If, instead, a Tuti Pattern is regarded as the final look of the fabric, we might have a better plan for the weaving matrix, as we show next.

Optimizing the Threading and Treadling

Taking a Tuti Pattern as the final visible pattern, the first thing to note is that evenly indexed rows and columns are dominated by 0's, and oddly indexed rows and columns are dominated by 1's. Specifically, all entries at $(2i, 2j)$ are 0's, and all at $(2i+1, 2j+1)$ are 1's. This makes it desirable — to avoid long floats — to use black yarns (or whatever color is chosen for 0) in evenly indexed rows and columns, and white yarns for oddly indexed ones. This designation of yarn colors is valid since at each intersection at least one of the two yarns bears the color of the final pattern.

With the aforementioned choice of yarn colors we gain a large degree of freedom (50% of the entries) to optimize the weaving matrix so as to control floats and reduce the number of shafts. Indeed, for all the black-black intersections at $(2i, 2j)$, and the white-white intersections at $(2i+1, 2j+1)$, it does not matter which yarn runs above or beneath. On the other hand, intersections at $(2i, 2j+1)$ and $(2i+1, 2j)$ dictate a specific designation of the weaving structure: yarns have different colors, and the yarn to pass above is the one whose color matches the final pattern at that point.

Let us now inspect the four distinct rows we enumerated earlier in the pattern, and see how they reflect in the weaving matrix, given our alternating choice of yarn colors. For evenly-indexed rows at $2i$ the sequence of colors $[0y_i0\bar{y}_i]$ dictates a weaving structure $[?\bar{y}_i?y_i]$, where "?" stands for "don't care" entries: either 0 or 1 would work. Note that the entries y_i and $\bar{y}_i$ in the color sequence are complemented in the weaving structure, because in these entries color 0 is supplied by the weft rather than the warp. For oddly-indexed rows the color sequence $(x_0 1 x_1 1 x_2 1 \cdots)$ at $4i+1$ dictates a weaving structure $(x_0 ? x_1 ? x_2 ? \cdots)$, and the color sequence $(\bar{x}_0 1 \bar{x}_1 1 \bar{x}_2 1 \cdots)$ at $4i+3$ dictates a weaving structure $(\bar{x}_0 ? \bar{x}_1 ? \bar{x}_2 ? \cdots)$. In matrix notation:

$$\mathbf{S} = \begin{pmatrix} ? & \bar{y}_0 & ? & y_0 & ? & \bar{y}_0 & \cdots \\ x_0 & ? & x_1 & ? & x_2 & ? & \cdots \\ ? & \bar{y}_1 & ? & y_1 & ? & \bar{y}_1 & \cdots \\ \bar{x}_0 & ? & \bar{x}_1 & ? & \bar{x}_2 & ? & \cdots \\ ? & \bar{y}_2 & ? & y_2 & ? & \bar{y}_2 & \cdots \\ x_0 & ? & x_1 & ? & x_2 & ? & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Now we make an important observation: the don't-care entries in evenly-indexed rows align with the governed entries in oddly-indexed rows, and vise-versa. Thus, we can merge each of the two distinct evenly-indexed rows, comprising alternating 0's and 1's every two steps, with one of the two distinct oddly-indexed rows, to obtain only two distinct rows in the weaving matrix. For example, we may use:

$$\mathbf{R_0} = (x_0 0 x_1 1 x_2 0 x_3 1 \cdots)$$

at rows $4i+1$, and:

$$\mathbf{R_1} = (\bar{x}_0 1 \bar{x}_1 0 \bar{x}_2 1 \bar{x}_3 0 \cdots)$$

at rows $4i+3$, and use $\mathbf{R_0}$ for rows $2i$ where $y_i = 1$, and $\mathbf{R_1}$ where $y_i = 0$. The complete weaving matrix becomes:

$$\mathbf{S} = \begin{pmatrix} x_0+\bar{y}_0 & \bar{y}_0 & x_1+\bar{y}_0 & y_0 & x_2+\bar{y}_0 & \bar{y}_0 & \cdots \\ x_0 & 0 & x_1 & 1 & x_2 & 0 & \cdots \\ x_0+\bar{y}_1 & \bar{y}_1 & x_1+\bar{y}_1 & y_1 & x_2+\bar{y}_1 & \bar{y}_1 & \cdots \\ \bar{x}_0 & 1 & \bar{x}_1 & 0 & \bar{x}_2 & 1 & \cdots \\ x_0+\bar{y}_2 & \bar{y}_2 & x_1+\bar{y}_2 & y_2 & x_2+\bar{y}_2 & \bar{y}_2 & \cdots \\ x_0 & 0 & x_1 & 1 & x_2 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Voila! The matrix is also made up of two distinct columns: $\mathbf{C_0} = (\bar{y}_0 0 \bar{y}_1 1 \bar{y}_2 0 \bar{y}_3 1 \cdots)'$ at columns $4j+1$, and $\mathbf{C_1} = (y_0 1 y_1 0 y_2 1 y_3 0 \cdots)'$ at columns $4j+3$. For evenly-indexed columns at $2j$ the value of x_j decides which column to use.

Thus, all Tuti Patterns are actually weave-able in looms with only two shafts! $\mathbf{R_0}$ becomes the threading sequence: to which shaft each warp yarn is attached, and $\mathbf{C_0}$ becomes the treadling sequence: which shaft is raised above each weft yarn. See Figure 3. How about floats? The presence of alternating 0's and 1's in all columns and rows controls the length of the floats: any four consecutive rows (analogously columns) contain both $\mathbf{R_0}$ and $\mathbf{R_1}$, and since these are the complements of each other, this structure fixes the maximum float lengths to 3: perfect! Note that this is the maximum, but not the expected float length, which would be 2 for random sequences $\mathbf{X}$ and $\mathbf{Y}$. It is worth noting that 2 is the float length in the ubiquitous twill fabrics.

More Resources, More Patterns

Similar to AA Outlines, Ahmed [3] recently presented an artwork at the Bridges exhbition that, instead of binary words, uses a pair of ternary words $\{\mathbf{X}, \mathbf{Y}\}$ to control the dashed lines; where a dash is two and a gap is one step long. Such a structure can also be rasterized to produce extended Tuti Patterns; see Figure 4. To weave these patterns we also assign alternating yarn colors to both warp and weft. The primary difference now is that $\mathbf{X}$ (analogously $\mathbf{Y}$) would distribute bits over 3 repeating rows. The weaving matrix now has

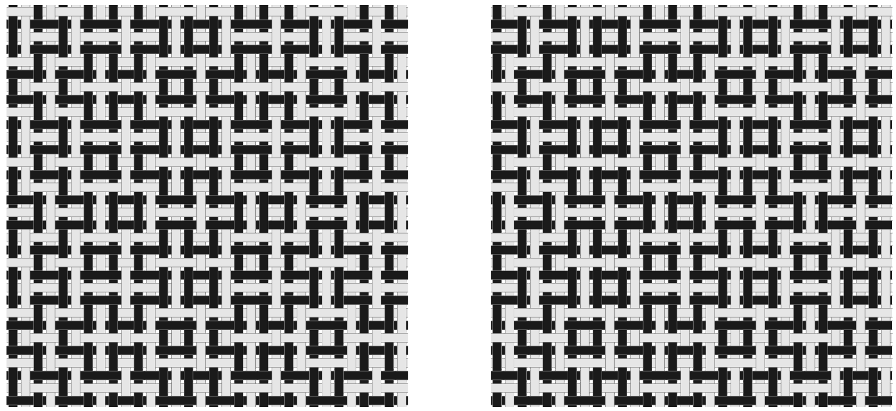


Figure 3 : *Front- and mirrored back-side of a weaving of the Tuti Pattern in Figure 2. It comprises only 2 distinct columns, and can therefore be woven in a 2-shaft loom. The two yarn colors are chosen to symbolize the Blue and White Niles, where "blue" actually means "dark" and "white" means "light" in Sudanese colloquial.*

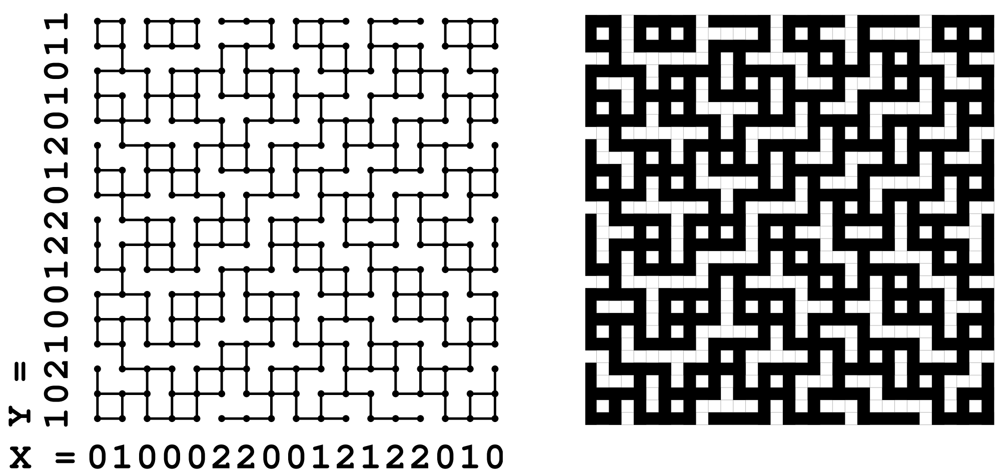


Figure 4 : *A "Boxes" structure controlled by two ternary words, and its rasterization into a Tuti Pattern.*

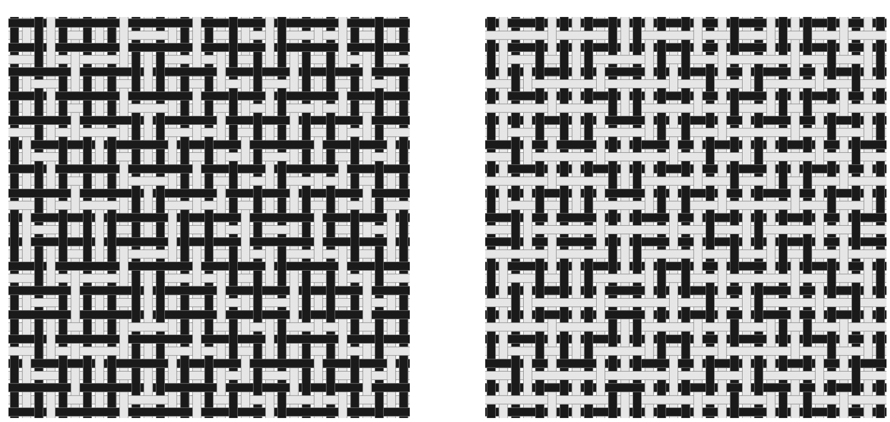


Figure 5 : *Front- and mirrored back-side of a weaving of the ternary Tuti Pattern in Figure 4. It comprises only 3 distinct columns, and can therefore be woven in a 3-shaft loom.*

three distinct columns. That is, a 3-shaft loom will suffice. The maximum float-length extends to five, which is still within the tolerable range. See Figure 5.

More generally, an n-ary AA Outline can be defined as the interaction of two orthogonal sets of dashed lines, each line has the same sequence of dashes

$$\mathbf{B} = [b_0 b_1 b_2 \cdots b_{n-1}] \ ,$$

where each b_i is either 0 (dash) or 1 (gap). The beginning of the dash-sequence in each line is determined by an n-ary digit read from the words $\{\mathbf{X}, \mathbf{Y}\}$ that control the outline. Any such outline can be rasterized into a Tuti Pattern and woven the same way as the binary counterpart. The don't-care entries in the weaving matrix:

$$\mathbf{S} = \begin{pmatrix} ? & \bar{b}_{y_0+0} & ? & \bar{b}_{y_0+1} & \cdots & \bar{b}_{y_0+n-1} & \cdots \\ b_{x_0+0} & ? & b_{x_1+0} & ? & \cdots & ? & \cdots \\ ? & \bar{b}_{y_1+0} & ? & \bar{b}_{y_1+1} & \cdots & \bar{b}_{y_1+n-1} & \cdots \\ b_{x_0+1} & ? & b_{x_1+1} & ? & \cdots & ? & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots \\ b_{x_0+n-1} & ? & b_{x_1+n-1} & ? & \cdots & ? & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

can be optimized:

$$\mathbf{S} = \begin{pmatrix} \bar{b}_{x_0+y_0} & \bar{b}_{y_0+0} & \bar{b}_{x_1+y_0} & \bar{b}_{y_0+1} & \cdots & \bar{b}_{y_0+n-1} & \cdots \\ b_{x_0+0} & b_0 & b_{x_1+0} & b_1 & \cdots & b_{n-1} & \cdots \\ \bar{b}_{x_0+y_1} & \bar{b}_{y_1+0} & \bar{b}_{x_1+y_1} & \bar{b}_{y_1+1} & \cdots & \bar{b}_{y_1+n-1} & \cdots \\ b_{x_0+1} & b_1 & b_{x_1+1} & b_2 & \cdots & b_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots \\ b_{x_0+n-1} & b_{n-1} & b_{x_1+n-1} & b_0 & \cdots & b_{n-2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

to make the matrix contain only n distinct columns and become weave-able in n-shaft looms. Note, however, that the matrix does not necessarily have n distinct rows. The threading sequence would be

$$R = x_0 0 x_1 1 \cdots x_{n-1}(n-1) x_n 0 x_{n+1} 1 \cdots \ .$$

The treadling sequence is replaced by a so-called "peg-plan": a sequence of n-bit binary words, where for each row each bit determines whether the respective shaft is (0) raised or (1) lowered. The sequence of each bit of the peg-plan corresponds to entries in a specific distinct column of the matrix $\mathbf{S}$. The maximum possible float is 1 step longer than twice the longest run in $\mathbf{B}$.

Unlike the binary Tuti Patterns, these extended patterns are not monotonous in tone: there are small variations in the color of the fabric that might suit some tastes, and the front and back side might reflect different tones. There is also a clear graph-theoretic difference: all vertices in binary AA Outlines have degree 2; that is, exactly two edges emerge from each vertex. Consequently, there are no T-junctions, line-breaks, or crosses in binary AA Outlines and Tuti Patterns; much like Op Art [11], but without the scintillating effect of long parallel lines. On the other hand, extended Tuti Patterns may contain crosses, T-junctions, and line breaks. The common feature of all Tuti Patterns, however, is that all lines are offset by a fixed distance, much like geometric Kufic Arabic calligraphy (see [12] for example).

Conclusion

This article demonstrated that a mathematical formulation could prove to be quite effective in manipulating classic problems in artistic contexts. It was not easy to guess that Tuti patterns could be woven in only 2-shaft looms, but the matrix formulation, along with the symbolic notation, made it easier to see such a potential.

Patterns derived for AA Patterns are good candidates for weaving designs mainly because they are constructed from two orthogonal structures, each made of a few distinct columns/rows. We still think that such structures are not exhausted, and we encourage research for more weave-able patterns. Another direction for future research is to study the different choices of the words $\{\mathbf{X}, \mathbf{Y}\}$ and their impact on the shape and the actual float lengths of Tuti Patterns.

Acknowledgments. Thanks to Ahmed Fuad and the anonymous reviewers for their helpful comments.

References

[1] Abdalla G. M. Ahmed. On the Fractal Behaviour of AA Patterns. In Hamish Carr and Silvester Czanner, editor, *Theory and Practice of Computer Graphics*, pages 93–97, Rutherford, United Kingdom, September 2012. Eurographics Association.

[2] Abdalla G. M. Ahmed. Modular Duotone Weaving Design. In Gary Greenfield, George Hart, and Reza Sarhangi, editors, *Proceedings of Bridges 2014: Mathematics, Music, Art, Architecture, Culture*, pages 27–34, Phoenix, Arizona, 2014. Tessellations Publishing. Available online at `http://archive.bridgesmathart.org/2014/bridges2014-27.html`.

[3] Abdalla G. M. Ahmed. Boxes. In Conan Chadbourne, Robert Fathauer, Katie McCallum, and Nathan Selikoff, editors, *Bridges Baltimore 2015 Art Exhibition Catalog*, page 3. Tessellations Publishing, Phoenix, Arizona, 2015.

[4] Abdalla G.M. Ahmed. AA Weaving. In George W. Hart and Reza Sarhangi, editors, *Proceedings of Bridges 2013: Mathematics, Music, Art, Architecture, Culture*, pages 263–270, Phoenix, Arizona, 2013. Tessellations Publishing. Available online at `http://archive.bridgesmathart.org/2013/bridges2013-263.html`.

[5] Sarah-Marie Belcastro. Reference Lists: Mathematical Articles on Fiber Arts. `http://www.toroidalsnark.net/mkreflist.html`, as of Jan. 27, 2016.

[6] Kris Bruland. Handweaving.net. `http://www.handweaving.net`, as of Feb 1, 2016.

[7] Ada K. Dietz. *Algebraic Expressions in Handwoven Textiles*. Little Loomhouse, 1949.

[8] Loe M. G. Feijs. Geometry and Computation of Houndstooth (Pied-de-poule). In Robert Bosch, Douglas McKenna, and Reza Sarhangi, editors, *Proceedings of Bridges 2012: Mathematics, Music, Art, Architecture, Culture*, pages 299–306, Phoenix, Arizona, USA, 2012. Tessellations Publishing.

[9] Andrew Glassner. Luminescence: Online Weaving Program. `https://www.imaginary-institute.com/resources/Luminescence/Luminescence.html`, as of Jan 27, 2016.

[10] Ralph E. Griswold. On-Line Digital Archive of Documents on Weaving and Related Topics. `http://www.cs.arizona.edu/patterns/weaving/webdocs.html`, as of Jan 27, 2016.

[11] Tiffany C. Inglis and Craig S. Kaplan. Generating Op Art Lines. In Douglas Cunningham and Tobias Isenberg, editors, *Workshop on Computational Aesthetics*, pages 25–32, Vancouver, Canada, 2011. Eurographics Association.

[12] Shakil Akram Khan. The Mathematician's Kite. In Conan Chadbourne, Robert Fathauer, and Katie McCallum, editors, *Bridges Seoul 2015 Art Exhibition Catalog*, page 66. Tessellations Publishing, Phoenix, Arizona, 2014.